GENERIC APPROACH TO ASSESSMENT OF NAVIGATIONAL RELIABILITY

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This paper presents the generic approach to the assessment of the navigational reliability in the specific sea way. This approach combines two methods that are usually used when studying the safety at sea problem: analytical and statistical ones. The expressions are obtained that may be used for calculating the probabilities of the collision and grounding.

Keywords: vessel traffic flow, safety at sea, navigational reliability, probability of a collision, probability of a grounding, random value, distribution function, density.

The trends of the world shipping development led to forming the intensive vessel traffic flows in many regions of the ocean. This stipulated the necessity of the development and introduction of the new methods of ensuring the safety at sea, for example the traffic separation schemes, the vessel traffic systems, ship's reporting systems and so on. To choose an optimal decision ensuring the safety at sea it is necessary to develop the method for assessing the navigational reliability in the congested waters. In addition the methods of such kind allow making a comparative analysis of the different decisions connected with the navigational reliability, the conditions of the navigation in the various regions and so on.

Two fundamental methods are usually used when studying the problem of safety at sea. First of them that may be named as statistical one is based on collection of long-term data on navigational situation in specific region and the following calculation of any indexes for example accidence index that is number of accidents referred to the total number of vessel passages during the determined time period. This method was used for example in the papers [1, 2].

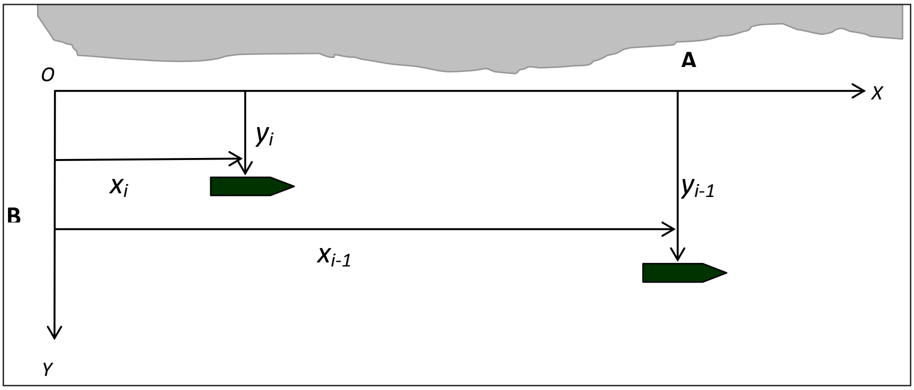
The second (analytical) method is concerned with the development of mathematical models in a varying degree simulating the real process of vessel traffic. There are many papers where the attempt was made to take into account many factors from the ship's speed and the passage width to the direction and velocity of the current and the tidal parameters [3-6]. But the models of this kind are very difficult to realize in practice.

This paper presents the approach to assessment of navigational reliability that combines both above-mentioned methods. On the basis of the generic laws of vessel traffic flow functioning the conditions that determine the accident occurring are analytically formulated. In general these conditions are expressed in the terms of probability distribution functions for the random values determining the vessel traffic parameters.

The derived expressions (analytical method) may be practically used only together with the preliminary collected and treated data (statistical method) on the navigational situation in the concrete area of the congested waters. It seems that this approach may be useful because it combines the advantages of both analytical and statistical methods.

It should be noted that in this paper the system of "traffic flow–sea way–navigators" is considered in the macro level without extracting the numerous separate factors influencing the functioning of this system. But it does not mean that these factors are not allowed: implicitly they exert influence on the form and parameters of above-mentioned probability distribution functions. So this approach is named as generic one. It may be presented by considering the following situation (see the Figure below).

Let us consider the vessel traffic flow that is moving on the sea way with length A and width B. A composition of the vessel traffic flow is characterized by the length L and velocity V of the vessels. A time interval between the passages over the section OY by two successive vessels is



denoted as T. A value reverse to the mean time interval t_m determines the intensity of this flow λ, i. e. $\lambda = 1/t_m$. In general a position of any vessel on the sea way is defined by a longitudinal distance X measured from the section OY and a transverse distance Y measured for example from the left side OX of the sea way. Additionally, the longitudinal and transverse distances between two vessels are denoted as D and Z. In this case L, V, X, Y, T, D and Z are the random values (RV) that have the distribution functions (DF) $F(l), F(v), F(x), F(y), F(t), F(d)$ and $F(z)$ and the probability density functions (PDF) $f(l), f(v), f(x), f(y), f(t), f(d)$ and $f(z)$ respectively.

A collision between two vessels may occur in two following conditions: (a) a dangerous approaching situation is taking place; (b) a navigator did not undertake the necessary actions to avoid collision or undertaken actions were not correct or adequate ones. The condition (a) may be formulated by using the known concept of the minimal allowable approaching distances: longitudinal (or dead water) one d_{min} and transverse (or beam) one z_{min}. As a simplification let d_{min} and z_{min} be the constant values. (It should be noted that these distances may be also considered as the longitudinal and transverse measures of the ship's domain). Then, the condition (a) is determined by joint occurring of the two events expressed in the form of inequalities $|Y_i - Y_{i-1}| = Z < z_{min}$ and $X_i - X_{i-1} = D < d_{min}$. In respect that D and Z are the RVs then the probabilities of occurring these two events are $P(Z < z_{min})$ and $P(D < d_{min})$. If the probability of occurring the condition (b) is denoted as P_b, then the collision of the two successive vessels on the sea way $P_{i,\ i-1}$ will occur with the probability as

$$P_{i,\ i-1} = P(Z < z_{min})\ P(D < d_{min})\ P_b. \qquad (1)$$

Let us consider the components of the expression (1). As Z is the RV then $P(Z < z_{min}) = F(z_{min})$. At the same time Z is the modulus of difference between Y_i and Y_{i-1}, i. e. $Z = |Y_i - Y_{i-1}|$, with Y is the RV with DF $F(y)$. In each particular case it is advisable to determine a form and parameters of the DF $F(y)$ on the base of collected statistic data. Nevertheless a general behavior of the traffic flows give grounds to suppose that the vessels across the sea way width are normally distributed. It is verified by several experimental studies [7]. Then if the RV Y_i and Y_{i-1} are normally distributed with the parameters m and σy, the difference of these value $Y_i - Y_{i-1}$ is also normally distributed with the parameters $m=0$ and $\sigma = \sqrt{2}\ \sigma_y$. As for the RV $Z = |Y|$ the DF and PDF are

$$f_z(z) = f_y(-z) + f_y(z) \qquad \text{for } z > 0$$

and $$F(z) = \int_0^z f(z)dz\ ,$$

then $$f_z(z) = \frac{1}{\sigma_y\sqrt{\pi}} \exp\left\{-\frac{z^2}{4\sigma_y}\right\} \qquad \text{for } z > 0 \qquad (2)$$

and $$F(z) = \frac{1}{\sigma_y\sqrt{\pi}} \int_0^z \exp\left\{-\frac{z^2}{4\sigma_y^2}\right\} dz = erf\left(\frac{z}{2\sigma_y}\right), \qquad (3)$$

where erf – a error function that is expressed in the general form as

$$erfx = \frac{e}{\sqrt{\pi}}\left(x - \frac{x^3}{3} + \frac{1}{2!}\frac{x^5}{5} - \frac{1}{3!}\frac{x^7}{7} + \cdots\right).$$

The second component of the expression (1) $P(D < d_{min})$ may be defined by two ways. First of them is to use the DF $F(d)$. If this function is known then $P(D < d_{min}) = F(d_{min})$. On the principle that the vessel traffic flow has a random nature [3, 5] it may be supposed that $F(d)$ is distributed by exponential law, i. e.

$$F(d) = 1 - \rho e^{-\rho d}\ , \qquad (4)$$

where ρ – a density of vessel traffic flow (it is a value reverse to the mean distance between two successive vessels, $\rho = 1/d_m$). The use of this way in each case requires a verification of the exponential nature of the *F(d)* or a determination of the effective form of this function distribution that makes it necessary to conduct a expensive location studies.

The second way of defining $P(D < d_{min})$ is to use an equivalent condition and other parameters characterizing the vessel traffic flow. Let us form the time condition providing the inequality $D < d_{min}$ is obeyed over the whole sea way length *A*. This condition is expressed as follows

$$\frac{A}{v_{i-1}} \prec \frac{A - d_{\min}}{v_i} + t_i \ , \tag{5}$$

where v_i and v_{i-1} are the velocities of the *i* and *i-1* vessels; t_i is the time interval between the *i* and *i-1* vessels.

Transforming the expression (5) gives

$$A \prec \frac{v_i v_{i-1} t_i - v_{i-1} d_{\min}}{v_i - v_{i-1}} \tag{6}$$

Let us introduce the new *RV* $S=(v_i v_{i-1} t_i - v_{i-1} d_{min})/(v_i - v_{i-1})$. If its DF *F(s)* is known then $P(D < d_{min}) = F(s=A)$.

As $F(s) = \int_0^s f(s)ds$ it is necessary to obtain the PDF *f(s)*.

It may be done by step-by-step transforming the expression $S=(v_i \ v_{i-1} \ t_i - v_{i-1} \ d_{min})/(v_i - v_{i-1})$ using following well-known equations of the probability theory for the RVs Y, X_1 and X_2

$$\begin{aligned} f(y) &= \int_{-\infty}^{\infty} f_2(x_2) f_1(y + x_2) dx_2 \quad \textit{for } Y = X_1 - X_2 \, , \\ f(y) &= \int_{-\infty}^{\infty} f_2(x_2) f_1\left(\frac{y}{x_2}\right)\left|\frac{1}{x_2}\right| dx_2 \quad \textit{for } Y = X_1 X_2 \, , \\ f(y) &= \int_{-\infty}^{\infty} f_2(x_2) f_1(y x_2) |x_2| dx_2 \quad \textit{for } Y = X_1 / X_2 \, . \end{aligned} \tag{7}$$

Accordingly a true form of *F(s)* and *f(s)* depends on the PDF*s* *f(v)* and *f(t)*. If the exponential distribution of the RV *T* is confirmed in some independent studies, for example [3, 5], then available data on the form of the PDF *f(v)* is rather different. So in the special cases the PDF follows the normal law [7], exponential law [7] or combined (uniform – directly proportional) law [3]. The hypothesis that RV *V* is distributed by the truncate-exponential, uniform or other laws is also worth notice. All the same using the equations (7) the correct analytic expression for the DF *F(s)* may by defined for any distribution of the RV *V* and *T*. At once it should be noted that the inequality (5) and accordingly the expression *F(s)* may be done more realistic by using not $d_{min}=const$, but $d_{min}= f(V, T)$. In this case the procedure of determining $P(D< d_{min})$ methodically is not changed but in some degree is complicated.

It is less obvious how to determine the third component of the expression (1) – the value P_b that defines the influence of a human factor. As the first approximation it may be used the value $P_b = 0.02$ that was obtained by the Japanese scientists on the base of processing the statistics. Certainly the separate wide studies are required with the assistance of the psychologists to obtain the more correct assessments of the value P_b.

As the expression (1) shows the product of the above considered components gives the probability of the collision between preceding vessel *i-1* and following vessel *i*, i. e. $P_{i, i-1}$. At the same time under some circumstances the probability exists that the vessel *i* will collide with the vessel $i – 2$ or vessel $i – 3$ and so on. These probabilities are $P_{i, i-2}$, $P_{i, i-3}$ and so on. They are defined in the same manner as $P_{i, i-1}$ saving that when calculating *F(d)* instead of *ρ* it should be used *ρ/2*, *ρ/3* and so on. Consequently, the total probability of collision for the vessel *i* on the sea way is

$$P_i = P_{i, i-1} + P_{i, i-2} + P_{i, i-3} + \ldots \qquad (8)$$

The number of components to be taken into account in the expression (8) depends on the specific conditions (the intensity or the density of the traffic flow, the length of the sea way and so on). In any case the numerical value of the probability $P_{i, i-j}$ is abruptly decreased.

So, the probability of the collision of any two vessels on the sea way of the length *A* during the specific time period that depends on the dimensions of *λ* is

$$P_{cl} = \sum_{i=\lambda} \sum_{i=\lambda-1} P_{i,i-j} = \lambda(P_{i,i-1} + P_{i,i-2} + \ldots) , \qquad (9)$$

where λ – the intensity of the vessel traffic flow transformed to the dimensions "number of vessels/time unit".

But the collision is not only navigational accident that is possible when sailing in the congested waters. Another accident is the grounding.

By analogy with the collision let us form the conditions leading to the grounding: (a) the transverse displacement of the vessel is reached or exceeded a dangerous value, i. e. $y_i \geq B$ and (b) a navigator did not undertake the necessary actions to avoid the grounding or undertaken actions were not correct or adequate ones. The probability of the condition (a) is $P(y \geq B) = 1 - F(y < B)$ and as in the first case the probability of occurring the condition (b) is denoted as P_b . Then the probability of the grounding of some specific vessel is $[1 - F(y)]\ P_b$ and the total probability of the grounding on the sea way during the time period that depends on the dimensions of λ is

$$P_{gr} = \lambda[1\text{-}F(y)]\ P_b\ . \tag{10}$$

It should be noted that the same method of determining P_{gr} with some variations (and sometimes for other purposes) is used in several previous papers of the other authors [1, 7].

If the method of determining the probability of navigational danger (collision and grounding) is available it is possible to evaluate the navigational reliability. For this purpose the navigational reliability may be treated as the functioning of the system "traffic flow – sea way – navigators" without an accident and may be assessed by the probability of zero navigational accidents on the given sea way during the specific time period that is

$$P = 1 - P_{ac} = 1\text{-}\ (\ P_{cl} + P_{gr})\ , \tag{11}$$

where P_{cl} and P_{gr} are calculated by the expressions (9) and (10).

Hereby, the approach considered allows obtaining the quantitative assessments of the reliability of the sailing without navigational accidents in the given system of "traffic flow – sea way - navigators". To do this it is necessary to know the statistical characteristics of the vessel traffic flows functioning in this area.

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